

# Matter Coupled $AdS_3$ Supergravities and Their Black Strings

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## Abstract

We couple  $n$  copies of  $N = (2, 0)$  scalar multiplets to a gauged  $N = (2, 0)$  supergravity in  $2 + 1$  dimensions which admits  $AdS_3$  as a vacuum. The scalar fields are charged under the gauged  $R$ -symmetry group  $U(1)$  and parametrize certain Kahler manifolds with compact or non-compact isometries. The radii of these manifolds are quantized in the compact case, but arbitrary otherwise. In the compact case, we find half-supersymmetry preserving and asymptotically Minkowskian black string solutions. For a particular value of the scalar manifold radius, the solution coincides with that of Horne and Horowitz found in the context of a string theory in  $2 + 1$  dimensions. In the non-compact case, we find half-supersymmetry preserving and asymptotically  $AdS_3$  string solutions which have naked singularities. We also obtain two distinct  $AdS_3$  supergravities coupled to  $n$  copies of  $N = (1, 0)$  scalar multiplets either by the truncation of the  $(2, 0)$  model or by a direct construction.

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# 1 Introduction

Important advances have been made in our understanding of  $M$ -theory in  $AdS$  space over the last few years [1, 2, 3]. In particular, evidence has been accumulated for a remarkable relation between certain gauged supergravities which admit  $AdS$  space as vacua and appropriate conformal field theories defined on the boundaries of these  $AdS$  spaces. An especially manageable example of this phenomenon arises in the context of  $AdS_3/CFT_2$  correspondence. While interesting work has been done on the CFT aspects of this problem, a great deal remains to be done on the supergravity side. With this motivation in mind, and in view of their relative simplicity, in this paper we study the structure of matter coupled  $AdS_3$  supergravity theories with  $N = (2, 0)$  and  $N = (1, 0)$  supersymmetry and their string solutions.

Since the  $AdS_3$  group is reducible, namely,  $SO(2, 2) \approx SO(2, 1)_I \times SO(2, 1)_{II}$ , the supersymmetry parameters could come in  $p$  copies of  $SO(2, 1)_I$  and  $q$  copies of  $SO(2, 1)_{II}$ , thus describing  $(p, q)$  supersymmetry. The pure  $AdS_3$  supergravity with  $(p, q)$  supersymmetry was constructed long ago and it is a Chern-Simons gauge theory based on the  $AdS_3$  supergroup  $OSp(p|1) \oplus OSp(q|1)$  [4]. Later, the coupling of the  $N = (2, 0)$  case to  $n$  copies of scalar multiplets which parametrize a Kahler manifold was constructed [5]. In this model, the scalars are neutral under the  $R$ -symmetry group  $U(1)$ . Supersymmetric solutions of this model have been studied and in particular, it has been shown [5] that the pure  $AdS_3$  supergravity sector of the theory admits the BTZ black hole [6] as a supersymmetric solution. The model has features unlike the familiar gauged supergravity theories. In fact, a higher dimensional origin of the model, whether it is field theoretic one or string/M-theoretic, is apparently not known.

There must exist, however, a class of (matter coupled)  $AdS_3$  supergravity theories which describe various  $AdS_3$  compactifications of string/M-theories. In particular, the  $AdS_3 \times S^3 \times K_3$  compactification of Type IIB string has been a subject of number of studies recently [9, 10, 11]. The full spectrum of this compactification is known, and the massless sector is expected to be described by a matter coupled  $N = (4, 4)$ ,  $AdS_3$  supergravity with the gauge group  $SO(3)_L \times SO(3)_R$ . There exist other supersymmetric compactifications of supergravities in diverse dimensions down to  $AdS_3$ , and in all these cases, we expect to find the gauged versions of the matter coupled Poincaré supergravities in  $2 + 1$  dimensions constructed long ago by Marcus and Schwarz [12], with the matter sector consisting of scalar multiplets with an underlying  $SO(8, n)/SO(n) \times SO(8)$  (for certain values of  $n$  implied by string theory) or  $E_8/SO(16)$  structure, and their lower supersymmetric truncations.

Ultimately we would like to construct all the  $AdS$  supergravities mentioned above in a unified framework, and to study their connection with the boundary conformal field theories. In this paper we take the first step in this direction. In particular, we construct an  $N = (2, 0)$ ,  $AdS_3$  supergravity coupled to  $n$ -complex dimensional Kahler manifold, and its  $N = (1, 0)$  truncation. This paper is devoted to understanding of the supergravity aspects of these models and their string solitons. We expect that there exist compactifications of string/M-theory giving rise to the supergravity theories studied here as their low energy limits.

As mentioned above there already exists a matter coupled  $N = (2, 0)$   $AdS_3$  supergravity constructed sometime ago by Izguierdo and Townsend [5]. However, this model differs from ours in

a significant way, namely its scalar fields are neutral with respect to the  $R$ -symmetry group  $U(1)$  unlike in our model. Consequently, the model of [5] does not have a potential while our model leads to a rather elaborate potential. In fact, some of the properties exhibited by our model are quite similar to those which arise in gauged  $N = 1$  supergravity coupled to a restricted kind of Kahler sigma model in  $D = 4$  [7]. For example, the sigma model manifold arising can be either compact or non-compact. The gravitational coupling constant, the radius of  $AdS_3$  and the radius of the sigma model manifold are not related to each other by supersymmetry, unlike the gauged supergravities with higher supersymmetries. Moreover, the radius of the sigma model manifold, when compact, is quantized in units of the gravitational coupling constant.

We also obtain a  $N = (1, 0)$  supersymmetric version of the results mentioned above by a consistent truncation of the  $N = (2, 0)$  model. We show that there exists a one parameter extension of this  $N = (1, 0)$  theory for the coupling of single scalar multiplet.

In this paper we also present string solutions of our models which preserve half supersymmetry, both for compact and non-compact sigma models. Interestingly enough, for the compact case there is a particular value of the radius such that the solution reduces to that of Horne and Horowitz [8] found in the context of low energy limit of a string theory in  $2 + 1$  dimensions. The solutions exhibit an event horizon and are asymptotically Minkowskian for the compact sigma model. The solutions for the non-compact sigma model, on the other hand, have naked singularities and are asymptotically  $AdS_3$ .

The plan of this paper is as follows. In Sec. 2, we describe the  $N = (2, 0)$   $AdS_3$  supergravity coupled to an  $n$ -complex dimensional Kahler sigma model. In Sec. 3, we specialize to the case of  $n = 1$ , namely  $CP^1$  and  $CH^1$ . The black string solutions and their properties will be discussed in Sec. 4. The  $N = (1, 0)$  supersymmetric matter coupled  $AdS_3$  supergravity is presented in Sec. 5. Our results and a number of open problems raised by them are discussed in Sec. 6.

## 2 $N=(2,0)$ $AdS_3$ Supergravity Coupled to $n$ Scalar Multiplets

The  $N = (2, 0)$   $AdS_3$  supergravity multiplet consists of a graviton  $e_\mu{}^a$ , two Majorana gravitini  $\psi_\mu$  (with the  $SO(2)$  spinor index suppressed) and an  $SO(2)$  gauge field  $A_\mu$ . The  $n$  copies of the  $N = (2, 0)$  scalar multiplet, on the other hand, consists of  $2n$  real scalar fields  $\phi^\alpha$  ( $\alpha = 1, \dots, 2n$ ) and  $2n$  Majorana fermions  $\lambda^r$  ( $r = 1, \dots, n$  and the  $SO(2)$  spinor indices are suppressed).

For simplicity, we will take the sigma model manifold  $M$  to be a coset space of the form  $G/H \times SO(2)$  where  $G$  can be compact or non-compact and  $H \times SO(2)$  is the maximal compact subgroup of  $G$ , where  $SO(2)$  is the  $R$ -symmetry group. For concreteness, we shall consider

$$M_+ = \frac{SO(n+2)}{SO(n) \times SO(2)}, \quad M_- = \frac{SO(n,2)}{SO(n) \times SO(2)}. \quad (2.1)$$

Our results can be readily translated to the case of  $G/H \times U(1)$  with  $G = SU(n+1)$  or  $SU(n,1)$  and  $H = SU(n)$ .

Let  $(L_I^i, L_I^r)$  where  $I = 1, \dots, n+2$ ,  $i = 1, 2$ ,  $r = 1, \dots, n$  form a representative of the coset  $M_\pm$ . It follows that

$$\begin{aligned} L_I^i L^{Ij} &= \pm \delta^{ij} , & L_I^r L^{Is} &= \delta^{rs} , & L_I^i L^{Ir} &= 0 , \\ \pm L_I^i L^{Ji} + L_I^r L^{Jr} &= \delta_I^J , \end{aligned} \quad (2.2)$$

where  $\pm$  correspond to the scalar manifolds  $M_\pm$ . The  $SO(n)$ ,  $SO(2)$  and  $SO(n+2)$  vector indices are raised and lowered with the Kronecker deltas and the  $SO(n, 2)$  vector indices with the metric  $\eta_{IJ} = \text{diag}(+ + \dots + --)$ .

The  $SO(2)$  gauged pull-back of the Maurer-Cartan form on  $M_\pm$  can be decomposed into the  $SO(n) \times SO(2)$  connections  $Q_\mu^{rs}$  and  $Q_\mu^{ij}$ , and the nonlinear covariant derivative  $P_\mu^{ir}$  as follows:

$$P_\mu^{ir} = (L^{-1} D_\mu L)^{ir} , \quad Q_\mu^{ij} = (L^{-1} D_\mu L)^{ij} , \quad Q_\mu^{rs} = (L^{-1} D_\mu L)^{rs} \quad (2.3)$$

where  $Q_\mu^{ij} \equiv Q_\mu \epsilon^{ij}$  and the  $SO(2)$  covariant derivative is defined as

$$D_\mu L = \left( \partial_\mu + \frac{1}{2} A_\mu^{ij} T_{ij} \right) L , \quad A_\mu^{ij} \equiv A_\mu \epsilon^{ij} . \quad (2.4)$$

The anti-hermitian  $SO(2)$  generator  $T_{ij}$  occurring in this definition is realized in terms of an  $(n+2) \times (n+2)$  matrix, which can be chosen as  $(T_{ij})_I^J = (\pm \delta_{Ii} \delta_j^J - i \leftrightarrow j)$ .

In coupling  $M_\pm$  to supergravity, we will also need the introduction of the “boosted matrix elements” defined as

$$\begin{aligned} \epsilon_{ij} \epsilon^{kl} C &= (L^{-1} T_{ij} L)^{kl} , \\ \epsilon_{ij} C^{rs} &= (L^{-1} T_{ij} L)^{rs} , \\ \epsilon_{ij} S^{kr} &= (L^{-1} T_{ij} L)^{kr} . \end{aligned} \quad (2.5)$$

From these definitions and (2.3) it follows that

$$\begin{aligned} \partial_{[\mu} Q_{\nu]} &= \frac{1}{2} \epsilon_{ij} P_\mu^{ir} P_\nu^{jr} + \frac{1}{2} F_{\mu\nu} C , \\ D_{[\mu} P_{\nu]}^{ir} &= \frac{1}{2} F_{\mu\nu} S^{ir} , \\ \partial_\mu C &= \epsilon_{ij} P_\mu^{ir} S^{jr} , \\ D_\mu C^{rs} &= \pm 2 P_\mu^{i[r} S^{s]i} , \\ D_\mu S^{ir} &= \pm \epsilon_{ij} P_\mu^{jr} C + C^{rs} P_\mu^{is} , \end{aligned} \quad (2.6)$$

where the covariant derivatives are defined as

$$\begin{aligned} D_\mu S^{ir} &:= \partial_\mu S^{ir} \pm Q_\mu^{ij} S^{jr} + Q_\mu^{rs} S^{is} , \\ D_\mu C^{rs} &:= \partial_\mu C^{rs} + Q_\mu^{rs} S^{is} . \end{aligned} \quad (2.7)$$

Recall that  $\pm$  correspond to the scalar manifolds  $M_\pm$  specified in (2.1).

Using the formulae given above and applying the standard Noether procedure we get the following matter coupled gauged supergravity Lagrangian up to quartic order fermion terms <sup>1</sup>

$$\begin{aligned} e^{-1} \mathcal{L} &= \frac{1}{4} R + \frac{1}{2} \epsilon^{\mu\nu\rho} \bar{\psi}_\mu D_\nu \psi_\rho - \frac{1}{16} \frac{e^{-1}}{ma^4} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - \frac{1}{4a^2} P_\mu^{ir} P_{ir}^\mu \\ &+ \frac{1}{2} \bar{\lambda}_r \gamma^\mu D_\mu \lambda^r + \frac{1}{2a} \bar{\lambda}_r \gamma^\mu \gamma^\nu \Gamma_i \psi_\mu P_\nu^{ir} - \frac{m}{2} \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu C^2 \\ &- 2ma \bar{\psi}_\mu \gamma^\mu \Gamma_i \Gamma^3 \lambda_r C S^{ir} - \frac{1}{2} m (1 \pm 4a^2) \bar{\lambda}^r \lambda_r C^2 \\ &+ 2ma^2 \bar{\lambda}_r \Gamma^3 \lambda_s C^{rs} C + 2ma^2 \bar{\lambda}_r \Gamma_i \Gamma_j \lambda_s S^{ir} S^{js} \\ &+ 2m^2 C^2 (C^2 - 2a^2 S^{ir} S_{ir}) , \end{aligned} \quad (2.8)$$

which has the local  $N = 2$  supersymmetry

$$\begin{aligned} \delta e_\mu^a &= -\bar{\varepsilon} \gamma^a \psi_\mu , \\ \delta \psi_\mu &= D_\mu \varepsilon + m \gamma_\mu C^2 \varepsilon , \\ \delta A_\mu &= 4ma^2 (\bar{\varepsilon} \Gamma^3 \psi_\mu) C - 4ma^3 (\bar{\lambda}_r \gamma_\mu \Gamma^i \varepsilon) S^{ir} , \\ L_i^I \delta L_I^r &= a \bar{\varepsilon} \Gamma_i \lambda^r , \\ \delta \lambda^r &= \left( -\frac{1}{2a} \gamma^\mu P_\mu^{ir} + 2ma \Gamma^3 C S^{ir} \right) \Gamma_i \varepsilon . \end{aligned} \quad (2.9)$$

We have set the gravitational coupling constant  $\kappa$  equal to one, but it can easily be introduced by dimensional analysis. The constant  $a$  is the characteristic curvature of  $M_\pm$  (e.g.  $2a$  is the inverse radius in the case of  $M_+ = S^2$ ) and the constant  $m$  is the  $AdS_3$  cosmological constant.

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<sup>1</sup>Our conventions are as follows:  $\eta_{ab} = (-++)$ ,  $\bar{\varepsilon} = \varepsilon^\dagger i \gamma_0$ ,  $\gamma^\mu C$  and  $\gamma^{\mu\nu} C$  are symmetric, the  $SO(2)$  charge conjugation matrix is unity,  $\Gamma^i$  is symmetric and  $\{\Gamma^i, \Gamma^j\} = 2\delta^{ij}$ . A convenient representation is  $\Gamma_1 = \sigma_1$ ,  $\Gamma_2 = \sigma_3$ . We define  $\Gamma_3 = \Gamma_1 \Gamma_2$ . Note that  $(\Gamma^3)^2 = -1$ .

The  $U(1)$  gauge coupling constant has been absorbed into the definition of  $A_\mu$ . We emphasize that, unlike in a typical anti de Sitter supergravity coupled to matter, here the constants  $\kappa, a, m$  are not related to each other for non-compact scalar manifolds, while  $a$  will be quantized in terms of  $\kappa$  in the compact case, as we shall see later.

The covariant derivatives are defined as

$$\begin{aligned} D_\mu \varepsilon &= \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} - \frac{1}{2a^2} Q_\mu \Gamma^3 \right) \varepsilon , \\ D_\mu \lambda^r &= \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} + \frac{1}{2a^2} Q_\mu \Gamma^3 \right) \lambda^r + Q_\mu{}^{rs} \lambda^s . \end{aligned} \quad (2.10)$$

The coefficients in front of the composite connection  $Q_\mu$  has been determined by supersymmetry.

Having defined the above covariant derivative, we can now see more clearly why the  $C$ - and  $S$ -functions arise in the model. Firstly, the supersymmetric variation of the gravitino kinetic term involves the commutator

$$[D_\mu, D_\nu] \varepsilon = \frac{1}{4} R_{\mu\nu ab} \gamma^{ab} \varepsilon - \frac{1}{2a^2} \left( \epsilon_{ij} P_\mu^{ir} P_\nu^{jr} + F_{\mu\nu} C \right) \Gamma^3 \varepsilon . \quad (2.11)$$

This is where we see first the occurrence of the function  $C$ . The  $C$ -dependent term arising here is cancelled by the variation of the Chern-Simons term. The rest of the Noether procedure eventually involves the differentiation of the function  $C$  which leads to the function  $S^{ir}$ , and its differentiation leads to the function  $C^{rs}$ .

It is straightforward to adapt all the formulae given above in terms of  $n$  complex scalars  $\phi^\alpha (\alpha = 1, \dots, n)$  and  $n$  Dirac spinors  $\lambda^r$  and a Dirac gravitino  $\psi_\mu$ , with the dreibein  $e_\mu^a$  and vector field  $A_\mu$ , of course, remaining real. Typical sigma model manifolds arising in this way are the compact  $CP^n$  and non-compact  $CH^n$ , if we are only concerned about the local aspects of the symmetries involved. Insisting that the model is globally well defined, some restrictions will arise on the scalar manifold geometry, as was shown long ago by Witten and Bagger in the context of  $N = 1$  supersymmetric models coupled to supergravity in  $D = 4$ . These restrictions typically occur when the scalar manifold is compact. Indeed, as in [7], here too the compact scalar manifold turns out to be a Hodge manifold, which is a certain type of Kahler manifold. An important consequence of this is that the radius of the scalar manifold gets quantized in units of the Planck length. This phenomenon is explained in detail in [7] and therefore will not be repeated here. However, we shall get back to the specifics of the quantization condition in the next section where we consider the black string solutions of the model when the scalar manifold is in effect taken to be  $S^2$ . In the case of  $H^2$  the quantization condition does not arise.

Let us consider various limits of this model. Firstly, by rescaling  $A_\mu \rightarrow a^2 A_\mu$  and the matter scalar fields such that  $P_\mu^{ir} \rightarrow a P_\mu^{ir}$  and then sending the inverse sigma model radius  $a \rightarrow 0$  such that  $C \rightarrow 1$ ,  $S^{ir} \rightarrow 0$ , one obtains  $N = (2, 0)$ ,  $AdS_3$  supergravity with cosmological constant

$-2m^2$  coupled to an  $R^{2n}$  sigma model. The pure  $N = (2, 0)$ ,  $AdS_3$  supergravity [4] can then be obtained by setting all the matter fields equal to zero. To obtain the Poincaré limit of the theory [12, 13, 14], on the other hand, we start with the Lagrangian (2.8), rescale  $A_\mu \rightarrow mA_\mu$  and then let  $m \rightarrow 0$ . Once the Poincaré limit is taken, the supergravity fields can further be decoupled by setting the gravitini equal to zero and taking the metric to be flat Minkowskian. This yields an  $N = (2, 0)$  supersymmetric sigma model. A rigid  $AdS_3$  supersymmetric limit does not seem to be possible in this model.

### 3 The Cases of $S^2$ and $H^2$

The variables of the model presented above can easily be complexified so that the scalar manifold becomes generically  $CP^n = SU(n+1)/SU(n) \times U(1)$  or  $CH^n = SU(n, 1)/SU(n) \times U(1)$ . In search of a string solution, it is convenient to set equal to zero all but one of the  $n$ -complex scalar fields in such a way that the model is consistently truncated to an  $S^2 = SU(2)/U(1)$  or  $H^2 = SU(1, 1)/U(1)$  sigma model coupled to the  $AdS_3$  supergravity with  $N = (2, 0)$  supersymmetry. The  $R^2$  geometry can easily be accounted for as the infinite radius limit of  $H^2$ .

The coset representative  $L$  for  $S^2$  and  $H^2$  can be parametrized as

$$L = \frac{1}{\sqrt{1 + \epsilon|\phi|^2}} \begin{pmatrix} 1 & \phi \\ -\epsilon\phi^\dagger & 1 \end{pmatrix}, \quad \epsilon = \begin{cases} +1 & \text{for } S^2 \\ -1 & \text{for } H^2 \end{cases} \quad (3.1)$$

Defining

$$\begin{aligned} SU(2) : \quad S &= S^1 + iS^2, \quad P_\mu = -iP_\mu^1 + P_\mu^2, \\ SU(1, 1) : \quad S &= -iS^1 + S^2, \quad P_\mu = P_\mu^1 + iP_\mu^2, \end{aligned} \quad (3.2)$$

the key relations (2.6) take the form

$$\begin{aligned} \partial_\mu C &= -\epsilon(P_\mu^* S + P_\mu S^*)/2, \\ D_\mu S &= P_\mu C. \end{aligned} \quad (3.3)$$

The functions  $C$  and  $S$  are computed from the definitions (2.5), which, for the cases at hand, are

$$L^{-1}T_3L = \frac{1}{2}(S_1 + iS_2)(T_1 - iT_2) + \frac{1}{2}(S_1 - iS_2)(T_1 + iT_2) + CT_3, \quad (3.4)$$

where the generators of  $SU(2)$  and  $SU(1, 1)$  algebras are

$$[T_1, T_2] = -T_3, \quad [T_2, T_3] = -\epsilon T_1, \quad [T_3, T_1] = -\epsilon T_2. \quad (3.5)$$

Representing  $(T_1, T_2, T_3)$  by  $(-i\sigma_1, i\sigma_2, -i\sigma_3)/2$  for  $SU(2)$  and  $(\sigma_1, -\sigma_2, i\sigma_3)/2$  for  $SU(1, 1)$ , we obtain from (3.4)

$$\begin{aligned} C &= \frac{1 - \epsilon|\phi|^2}{1 + \epsilon|\phi|^2}, \\ S &= \frac{2\phi}{1 + \epsilon|\phi|^2}. \end{aligned} \quad (3.6)$$

Similarly, the nonlinear covariant derivative  $P_\mu$  and the  $SO(2)$  connection  $Q_\mu$  are computed from the definitions (2.3), which for the cases we are considering, take the form

$$L^{-1}(\partial_\mu + A_\mu T_3)L = \frac{1}{2}(P_\mu^1 + iP_\mu^2)(T_1 - iT_2) + \frac{1}{2}(P_\mu^1 - iP_\mu^2)(T_1 + iT_2) + Q_\mu T_3, \quad (3.7)$$

from which, recalling the definitions (3.2), it follows that

$$\begin{aligned} P_\mu &= \frac{2D_\mu\phi}{1 + \epsilon|\phi|^2} \\ &= \frac{2\partial_\mu\phi}{1 + \epsilon|\phi|^2} - i\epsilon A_\mu S, \\ Q_\mu &= \frac{i\phi \overset{\leftrightarrow}{D}_\mu \phi^*}{1 + \epsilon|\phi|^2} + A_\mu \\ &= \frac{i\phi \overset{\leftrightarrow}{\partial}_\mu \phi^*}{1 + \epsilon|\phi|^2} + A_\mu C, \end{aligned} \quad (3.8)$$

where

$$D_\mu\phi = (\partial_\mu - i\epsilon A_\mu)\phi. \quad (3.9)$$

In describing the sigma model manifolds, we have used a particular coordinate system. We have to ensure that this description makes sense globally. In fact, the coordinates  $\phi$  are the stereographic projection of  $S^2$  or  $H^2$  onto the complex plane. In the case of  $H^2$  this is a globally well defined coordinate system to cover the manifold. But in the case of  $S^2$ , as is well known, one needs two patches in order to avoid the singularities at the north and south poles. Following the standard procedure, we cover the upper hemisphere with coordinate  $\phi$  and the lower one by  $1/\phi$ . We must then check that the action is well defined in the overlap region. To achieve this, we also need to transform the gauge field as  $A_\mu \rightarrow -A_\mu$ . Under the combined transformation



$$\phi \rightarrow \frac{1}{\phi} , \quad A_\mu \rightarrow -A_\mu , \quad (3.10)$$

the quantities  $C, S, P_\mu$  and  $Q_\mu$  transform as

$$\begin{aligned} C &\rightarrow -C , \\ S &\rightarrow \left( \frac{\phi^*}{\phi} \right) S , \\ P_\mu &\rightarrow - \left( \frac{\phi^*}{\phi} \right) P_\mu , \\ Q_\mu &\rightarrow Q_\mu + i \partial_\mu (\ln \phi - \ln \phi^*) . \end{aligned} \quad (3.11)$$

For these transformations to leave the action invariant, we must also transform the fermionic fields. Noting that these terms are given by

$$\frac{1}{2} \epsilon^{\mu\nu\rho} \bar{\psi}_\mu \left( \nabla_\nu - \frac{i\kappa^2}{2a^2} Q_\nu \right) \psi_\rho + \frac{1}{2} \bar{\lambda} \gamma^\mu \left( \nabla_\mu - \frac{i\kappa^2}{2a^2} Q_\mu \right) \lambda , \quad (3.12)$$

where  $\nabla_\mu$  is the Lorentz covariant derivative, we find that the appropriate transformation rules for the fermions are

$$\begin{aligned} \psi_\mu &\rightarrow \exp \left[ -\frac{\kappa^2}{2a^2} (\ln \phi - \ln \phi^*) \right] \psi_\mu , \\ \lambda &\rightarrow \exp \left[ -\frac{\kappa^2}{2a^2} (\ln \phi - \ln \phi^*) \right] \lambda , \end{aligned} \quad (3.13)$$

where we have re-introduced the gravitational coupling constant  $\kappa$ . For these transformations to be single valued, we need to impose, á la Witten and Bagger [7], the quantization condition

$$\frac{\kappa^2}{a^2} = n , \quad (3.14)$$

where  $n$  is an integer.

## 4 The Black String Solution

We shall now seek string solutions of the model described in the previous section. To this end, let us note the bosonic part of the Lagrangian

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{1}{16}\frac{e^{-1}}{ma^4}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{|D_\mu\phi|^2}{a^2(1+\epsilon|\phi|^2)^2} - V(\phi) , \quad (4.1)$$

where the potential is given by

$$V(\phi) = 4m^2a^2C^2\left(|S|^2 - \frac{1}{2a^2}C^2\right) , \quad (4.2)$$

and  $C$  and  $S$  are defined in (3.6). Note that  $\epsilon|S|^2 = 1 - C^2$ . The resulting bosonic field equations are

$$R_{\mu\nu} = \frac{1}{a^2}P_{(\mu}P_{\nu)}^* + 4Vg_{\mu\nu} , \quad (4.3)$$

$$\sqrt{-g}\epsilon_{\mu\nu\rho}F^{\nu\rho} = -4ima^2(P_\mu S^* - P_\mu^* S) , \quad (4.4)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}P_\nu) - i\epsilon Q_\mu P^\mu = 2a^2(1 + \epsilon|\phi|^2)\frac{\partial V}{\partial\phi^*} \quad (4.5)$$

where

$$\frac{\partial V}{\partial\phi} = -\frac{8\epsilon m^2}{1 + \epsilon|\phi|^2}\left(a^2|S|^2 - (1 + \epsilon a^2)C^2\right)CS^* . \quad (4.6)$$

We shall also need the supersymmetry transformation rules. Let us first define

$$\hat{\varepsilon} := \frac{1}{2}(1 - i\Gamma_3)\varepsilon , \quad \hat{\psi}_\mu := \frac{1}{2}(1 - i\Gamma_3)\psi_\mu , \quad \hat{\lambda} := \frac{1}{2}(i)^{(1+\epsilon)/2}(1 + i\Gamma_3)\lambda . \quad (4.7)$$

Dropping the hat for notational convenience, the transformation rules (2.9), applied to the case at hand, take the form

$$\begin{aligned} \delta\psi_\mu &= D_\mu\varepsilon + m\gamma_\mu C^2\varepsilon , \\ \delta\lambda &= \left(-\frac{1}{2a}\gamma^\mu P_\mu + 2\epsilon ma CS\right)\Gamma^-\varepsilon , \end{aligned} \quad (4.8)$$

where  $\Gamma^\pm = (\Gamma^1 \pm i\Gamma^2)/2$ .

Before presenting the black string solutions, it is worthwhile to note that the theory admits various maximally symmetric vacua. For the case of  $S^2$ , the potential (4.2) has minimum at  $\phi = 0$  corresponding to a supersymmetric  $AdS_3$  vacuum, a valley of minima at  $\phi = e^{i\theta}$  corresponding to a supersymmetric 2 + 1 dimensional Minkowski vacuum and two valleys of maxima at  $\phi_{\pm} = (1 \mp \lambda/1 \pm \lambda)^{1/2} e^{i\theta}$ , where  $\lambda = a/\sqrt{2a^2 + 1}$ , corresponding to non-supersymmetric de Sitter vacua. Here  $\theta$  is an arbitrary real scalar field. For the case of  $H^2$ , we have the following extrema: (i) For  $a^2 \leq 1/2$ , there is a maximum at  $\phi = 0$  which is a supersymmetric  $AdS_3$  vacuum, (ii) for  $1/2 < a^2 < 1$ , there are two valleys of minima at  $\phi_{\pm} = (\pm\lambda + 1/\pm\lambda - 1)^{1/2} e^{i\theta}$  where  $\lambda = a/\sqrt{2a^2 - 1}$  which are non-supersymmetric  $AdS_3$  vacua, (iii) for  $a^2 \geq 1$  there is a minimum at  $\phi = 0$  which is a supersymmetric  $AdS_3$  vacuum. The case (ii) similar in nature to a situation encountered in finding the extrema of the gauged  $D = 7$  supergravity theory [15].

Let us now consider the following ansatz for the metric

$$ds^2 = e^{2A}(-dt^2 + dx^2) + e^{2B}dr^2, \quad (4.9)$$

where  $A, B$  are functions of the transverse coordinate  $r$  only. Next, we set

$$\phi = |\phi|, \quad A_{\mu} = 0, \quad \psi_{\mu} = 0, \quad \lambda = 0. \quad (4.10)$$

Then, the supersymmetry condition  $\delta\lambda = 0$  implies that

$$e^{-B}\phi' = 4ma^2C\phi, \quad (4.11)$$

where the prime indicates differentiation with respect to  $r$ , provided that we also impose the condition

$$\gamma^1\varepsilon = -\epsilon\varepsilon, \quad (4.12)$$

which means that we are seeking half-supersymmetry preserving solution. The choice of minus sign is merely for convenience.

The supersymmetry conditions  $\delta\psi_0 = 0$  and  $\delta\psi_2 = 0$  are satisfied provided that

$$A' = 2\epsilon m C^2 e^B. \quad (4.13)$$

The remaining condition  $\delta\psi_1 = 0$  determines the  $r$ -dependence of the spinor  $\varepsilon$  to be

$$\varepsilon = S^{1/4a^2} (1 - \epsilon\gamma_1)\varepsilon_0, \quad (4.14)$$

where  $\varepsilon_0$  is an arbitrary constant spinor. Next, we use (4.13) in (4.11), and solve for  $A$  in terms of  $\phi$ :

$$e^A = \left( \frac{2\phi}{1 + \epsilon\phi^2} \right)^{\frac{\epsilon}{2a^2}}, \quad (4.15)$$

where we have set a multiplicative integration constant equal to 2 for convenience. Thus, the metric takes the form

$$ds^2 = \left( \frac{2\phi}{1 + \epsilon\phi^2} \right)^{\epsilon/a^2} (-dt^2 + dx^2) + \frac{1}{16m^2a^4} \left( \frac{1 + \epsilon\phi^2}{1 - \epsilon\phi^2} \right)^2 \left( \frac{\phi'}{\phi} \right)^2 dr^2 \quad (4.16)$$

It is straightforward to verify that all the field equations are satisfied by this metric and the ansatz (4.9).

The fact that  $\phi$  is not determined by the equations of motion is a consequence of having freedom in reparametrizing the radial coordinate  $r$ . Indeed, the function  $\phi$  can be determined by performing a  $\phi$ -dependent  $r$ -coordinate transformation. A convenient such transformation is

$$r \rightarrow \tilde{r} = M \left( \frac{1 + \phi^2(r)}{1 - \phi^2(r)} \right)^2, \quad (4.17)$$

where  $M$  is an integration constant. We next analyze the compact and non-compact cases separately.

#### 4.1 The Case of $H^2$ ( $\epsilon = -1$ )

The inversion of (4.17) yields the  $\tilde{r}$ -dependence of  $\phi$

$$\phi = \left( \frac{\sqrt{\tilde{r}} - \sqrt{M}}{\sqrt{\tilde{r}} + \sqrt{M}} \right)^{1/2}. \quad (4.18)$$

where the tilde on  $r$  has been dropped for notational convenience. In obtaining this result, we have chosen the positive root in (4.17). The negative root gives an expression for  $\phi$  which diverges at  $r = M$ . Note that  $r \geq M$  implies  $\phi \geq 0$  in accordance with the fact that  $\phi$  is the stereographic coordinate of  $H^2$ .

The transformation (4.17) turns the metric (4.16) into

$$ds^2 = \left( \frac{r}{M} - 1 \right)^{\frac{-1}{2a^2}} (-dt^2 + dx^2) + \frac{1}{64m^2a^4r^2} \left( \frac{r}{M} - 1 \right)^{-2} dr^2. \quad (4.19)$$

This metric has no horizons and there is a naked singularity. To see this, we first let  $r \rightarrow r + M$  and then define a new radial coordinate  $u = M/r$ . The metric then takes the form

$$ds^2 = (u)^{\frac{1}{2a^2}} (-dt^2 + dx^2) + \frac{1}{64m^2a^4(u+1)^2} du^2. \quad (4.20)$$

The asymptotic geometry near  $u = \infty$  is  $AdS_3$ . The metric has a singularity at  $u = 0$ , while it is regular at other points. The fact that  $u = 0$  is a genuine singularity can be seen from the curvature scalar associated with this metric, given by

$$R = \frac{8m^2(u+1)}{u^2} \left[ 8a^2 - 3(u+1) \right], \quad (4.21)$$

which clearly diverges for  $u = 0$ . The implications of this naked singularity for the cosmic censorship conjecture remains to be investigated.

Finally, we find that the  $AdS$  energy per unit length for the string metric (4.20) vanishes. Actually, the commutator of two  $N = (2, 0)$  supersymmetry transformations can be shown to vanish at radial infinity, but one cannot deduce from this alone that the  $AdS$  energy vanishes. This is due to the fact that the result is a combination of the true Lorentz rotations and translations in  $AdS_3$ . A more convenient method to pin down the  $AdS$  energy for the case at hand is due to Hawking and Horowitz [16], and applying this method, we indeed find the result stated above, namely the vanishing of the  $AdS$  energy for our solution.

## 4.2 The Case of $S^2$ ( $\epsilon = 1$ )

The inversion of (4.17) for the upper hemisphere  $S_+^2$  yields

$$\phi_+ = \left( \frac{\sqrt{r} - \sqrt{M}}{\sqrt{r} + \sqrt{M}} \right)^{1/2}. \quad (4.22)$$

For the lower hemisphere  $S_-^2$  we obtain

$$\phi_- = \left( \frac{\sqrt{r} + \sqrt{M}}{\sqrt{r} - \sqrt{M}} \right)^{1/2}. \quad (4.23)$$

Note that  $r \geq M$  in both cases, in accordance with the fact that  $\phi_{\pm}$  are the stereographic coordinates of  $S^2$  such that  $0 \leq \phi_+ \leq 1$  and  $1 \leq \phi_- < \infty$ . In fact,  $\phi_+$  and  $\phi_-$  constitute a well defined map from spacetime into  $S^2$ .

With the scalar field specified as in (4.22) or (4.23), the metric (4.16) becomes

$$ds^2 = \left( 1 - \frac{M}{r} \right)^{\frac{1}{2a^2}} (-dt^2 + dx^2) + \frac{1}{64m^2a^4r^2} \left( 1 - \frac{M}{r} \right)^{-2} dr^2. \quad (4.24)$$

Note that for this case  $1/a^2$  is quantized to be an integer. This metric is asymptotically Minkowskian. Moreover, there is a horizon at  $r = M$ , and the near horizon geometry is  $AdS_3$ . The Hawking temperature of this black string can be readily shown to be vanishing. Thus, we expect this solution to be quantum mechanically stable.

The curvature scalar associated with the metric (4.24) is

$$R = \frac{64m^2a^2}{r^2} \left[ Mr - M^2 \left( 1 + \frac{3}{8a^2} \right) \right] \quad (4.25)$$

which is regular at  $r = M$ . This formula also shows that there is a singularity at  $r = 0$ . However, for some values of the parameter  $a$  the singularity cannot be reached by the observers outside the horizon. To investigate this point, let us consider the geodesic equation. Let  $\xi^\mu$  be tangent to an affinely parametrized geodesic, and let us define the conserved quantities associated with the two translations on the string worldsheet as  $E = -\xi \cdot \partial/\partial t$ ,  $P = \xi \cdot \partial/\partial x$ . Then the geodesic equation associated with the metric (4.24) takes the form

$$\frac{1}{64m^2a^4} \left( \frac{\dot{r}}{r} \right)^2 = \alpha \left( 1 - \frac{M}{r} \right)^2 + (E^2 - P^2) \left( 1 - \frac{M}{r} \right)^{(4a^2-1)/2a^2}, \quad (4.26)$$

where the dot denotes derivative with respect to an affine parameter,  $\alpha = 0$  for null geodesics and  $\alpha = -1$  for timelike geodesics. For  $\alpha = -1$ , the geodesics can not reach the horizon. Indeed, there is a turning point corresponding to the vanishing of the right hand side of (4.26). For  $\alpha = 0$ , a simple analysis of (4.26) near the horizon shows that when  $1/2a^2$  is an even integer the region  $r < M$  is accessible, but not accessible when  $1/2a^2$  is an odd or half integer. For the former case, we need to extend the definition of  $\phi$  to the region  $r < M$ . However, Einstein equations imply that  $C^2 = M/r$  and thus  $C > 1$  for  $r < M$ . On the other hand, we see from its definition that  $C^2 \leq 1$  for any value of  $\phi$ . Therefore, we can not extend the solution to the region  $r < M$  when  $1/2a^2$  is an even integer.

To summarize, we have physically well defined black string solutions for

$$\frac{1}{a^2} = 1, 2, 3 \bmod 4. \quad (4.27)$$

In these cases the timelike or null geodesics can not penetrate the horizon located at  $r = M$ , and the field  $\phi$  need not be extended to the region  $r < M$ .

For  $a^2 = 1/2$ , the metric (4.24) coincides with the metric found by Horne and Horowitz [8] obtained from a different starting point, namely low energy limit of a string theory in  $2+1$  dimensions described by the Lagrangian

$$e^{-1}\mathcal{L} = e^\phi \left( R + \partial_\mu \phi \partial^\mu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{8}{k} \right), \quad (4.28)$$

where  $H = dB$  and  $k$  is a constant. In Einstein frame, this Lagrangian takes the form

$$e^{-1}\mathcal{L} = R - \partial_\mu\phi\partial^\mu\phi - \frac{1}{2}e^{4\phi}H^2 + \frac{8}{k}e^{-2\phi} . \quad (4.29)$$

The metric (4.16) is a solution for this theory, with the dilaton given by  $\phi = \ln(r\sqrt{k/2})$ . What we have shown here is that not only this metric is a solution of two rather different theories but it is also supersymmetric. We note that the string theory which should generate our matter coupled  $N = (2, 0)$   $AdS_3$  supergravity model remains to be determined.

Finally, we note that the mass per unit length for our general string solutions in the  $S^2$  sigma model case can be conveniently deduced from the algebra of supercharges, since these solutions are asymptotically Minkowskian. A standard procedure which makes use of the Nester two-form (see, for example, [17, 18]) yields the result

$$\begin{aligned} [Q_{\varepsilon_1}, Q_{\varepsilon_2}] &= \lim_{r \rightarrow \infty} \frac{\epsilon^{012}}{\sqrt{-g}} \bar{\varepsilon}_1 \left( D_2 + m\gamma_2 C^2 \right) \varepsilon_2 \\ &= \bar{\varepsilon}_1 \left( \gamma^0 P_0 + \gamma^2 P_2 \right) \varepsilon_2 , \end{aligned} \quad (4.30)$$

where  $P_0 = P_2 = 8m^2 a^2 M$ , and 0, 1, 2 refer to the time, radial and  $x$ -directions, respectively, in coordinate basis ( $\epsilon^{012} = 1$ , in our conventions). Thus, the string has mass and momentum per unit length  $8m^2 a^2 M$ . We refer the reader to [19] for a study of various aspects of this phenomenon.

## 5 The $N = (1, 0)$ $AdS_3$ Truncation

The  $N = (2, 0)$  model described in the previous sections admits a truncation to  $N = (1, 0)$  supersymmetric  $AdS_3$  supergravity coupled to  $n$  scalar multiplets. It is straightforward to check that the following truncation is consistent:

$$\begin{aligned} A_\mu &= 0 , \quad P_\mu^{1r} = 0 , \\ (1 - \Gamma_2)\psi_\mu &= 0 , \quad (1 - \Gamma_2)\lambda^r = 0 , \quad (1 - \Gamma_2)\varepsilon = 0 . \end{aligned} \quad (5.1)$$

The condition  $P_\mu^{1r} = 0$  amounts to setting  $n$  of the original  $2n$  scalar fields equal to zero. A convenient way to realize this condition is to parametrize  $L$  as follows

$$L = \begin{pmatrix} (I - \epsilon\phi\phi^T)^{1/2} & 0 & \phi \\ 0 & 1 & 0 \\ -\epsilon\phi^T & 0 & (1 - \epsilon\phi^T\phi)^2 \end{pmatrix} , \quad (5.2)$$

where  $\phi$  is an  $n$ -component column vector representing the  $n$  real scalars. From the definitions (2.5) and the representation  $(T_{ij})_I^J = (\epsilon \delta_{Ii} \delta_j^J - i \leftrightarrow j)$ , it follows that

$$C^{rs} = 0, \quad S^{2r} = 0, \quad Q_\mu = 0, \quad (5.3)$$

and

$$\begin{aligned} C &= (1 - \epsilon \phi^2)^{1/2}, \\ S^r &= \phi^r, \\ P_\mu^r &= \epsilon \left[ (1 - \epsilon \phi \phi^T)^{-1/2} \partial_\mu \phi \right]^r, \\ Q_\mu^{rs} &= 2\phi^{-2} \left[ 1 - (1 - \epsilon \phi^2)^{1/2} \right] \phi^{[r} \partial_\mu \phi^{s]}, \end{aligned} \quad (5.4)$$

where  $\phi^2 \equiv \phi^T \phi$ ,  $S^r \equiv S^{1r}$  and  $P_\mu^r \equiv P_\mu^{2r}$ . The identities (2.6) now take the form

$$\partial_\mu C = -P_\mu^r S^r, \quad D_\mu S^r = \epsilon P_\mu^r C. \quad (5.5)$$

Performing the truncation procedure described above, we are left with the  $N = (1, 0)$   $AdS_3$  supergravity multiplet consisting of a dreibein and a single Majorana gravitino, and  $n$  copies of  $N = (1, 0)$  scalar multiplets each one of which contain a real scalar and a Majorana spinor. The generic manifolds parametrized by the scalar fields are now

$$N_+ = \frac{SO(n+1)}{SO(n)}, \quad N_- = \frac{SO(n, 1)}{SO(n)}. \quad (5.6)$$

The truncation of the Lagrangian (2.8) gives

$$\begin{aligned} e^{-1} \mathcal{L} &= \frac{1}{4} R + \frac{1}{2} \epsilon^{\mu\nu\rho} \bar{\psi}_\mu D_\nu \psi_\rho - \frac{1}{4a^2} P_\mu^r P_r^\mu \\ &+ \frac{1}{2} \bar{\lambda}_r \gamma^\mu D_\mu \lambda^r + \frac{1}{2a} \bar{\lambda}_r \gamma^\mu \gamma^\nu \psi_\mu P_\nu^r - \frac{m}{2} \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu C^2 \\ &- 2ma \bar{\psi}_\mu \gamma^\mu \lambda_r C S^r - \frac{1}{2} m (1 \pm 4a^2) \bar{\lambda}^r \lambda_r C^2 \\ &+ 2ma^2 \bar{\lambda}_r \lambda_s S^r S^s + 2m^2 C^2 (C^2 - 2a^2 S^r S^r), \end{aligned} \quad (5.7)$$

which has the local  $N = (1, 0)$  supersymmetry



$$\begin{aligned}
\delta e_\mu^a &= -\bar{\varepsilon}\gamma^a\psi_\mu , \\
\delta\psi_\mu &= D_\mu\varepsilon + m\gamma_\mu C^2\varepsilon , \\
L^I\delta L_I^r &= a\bar{\varepsilon}\lambda^r , \\
\delta\lambda^r &= \left(-\frac{1}{2a}\gamma^\mu P_\mu^r - 2maCS^r\right)\varepsilon .
\end{aligned} \tag{5.8}$$

The index  $I$  now labels the  $n+1$  dimensional representation of  $SO(n+1)$  or  $SO(n,1)$ , and the matrices  $(L^I, L_r^I)$  together form an element of these groups. The latter can be represented by (5.2) with the  $(n+1)$ 'st row and column deleted.

Clearly the black string solution of the  $N = (2, 0)$  model described in Section 4 is also a solution of the  $N = (1, 0)$  model given here.

The  $C$ - and  $S$ -functions defined in (2.6) arose as a consequence of the commutator (2.11). It is noteworthy that these functions still arise in the  $N = (1, 0)$  model despite the fact that the commutator (2.11) no longer occurs. Indeed, as far as supersymmetry is concerned, all that is required of the  $C$  and  $S$ -functions is that they obey the relations (5.5). This suggests the possibility of a more general solution for them. To see this, let us consider the case of  $SO(1, 1)$  scalar manifold. In that case the  $C$ - and  $S$ -functions take the simple form

$$C = \cosh \varphi , \quad S = \sinh \varphi , \tag{5.9}$$

where we have defined  $\phi = \sinh \varphi$ . The bosonic Lagrangian then becomes

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{1}{4a^2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) , \tag{5.10}$$

where

$$V = 2m^2\cosh^2\varphi \left(2a^2\sinh^2\varphi - \cosh^2\varphi\right) . \tag{5.11}$$

A more general solution of the defining relation (5.5) is

$$C = a_1e^\varphi + a_2e^{-\varphi} , \quad S = -a_1e^\varphi + a_2e^{-\varphi} , \tag{5.12}$$

where  $a_1, a_2$  are arbitrary real constants. These functions define a family of  $N = (1, 0)$   $AdS_3$  coupled to a single scalar multiplet, with potential

$$V(\varphi) = 4m^2a^2 \left[ (a_1e^\varphi + a_2e^{-\varphi})^4 - \frac{1}{2a^2} (a_1^2e^{2\varphi} - a_2^2e^{-2\varphi})^2 \right] . \tag{5.13}$$

In fact,  $a_1/a_2$  is the only independent parameter, due to the freedom in rescaling the parameter  $m$ . For  $a_1/a_2 = 1$ , one obtains the  $N = (1, 0)$  truncation of the  $N = (2, 0)$  model discussed above.

## 6 Conclusions

We have constructed the coupling of  $n$ -complex dimensional Kahler sigma models of certain type to the  $AdS_3$  supergravity with  $N = (2, 0)$  supersymmetry and we have obtained the black string solutions of this model. We have also obtained the  $N = (1, 0)$  truncation of our model, which still admits a potential as well as the solutions of the  $N = (2, 0)$  model discussed here.

Our models generically depend on two parameters which characterize the sizes of the  $AdS_3$  and the sigma model manifold, respectively. Moreover, the geometry of the sigma models can be compact or non-compact. The properties of the string solutions presented here depend on the geometry of the sigma model. In the compact case, we have found asymptotically Minkowskian black string solutions, while in the non-compact case we have found asymptotically  $AdS_3$  string solutions with naked singularities. In the former case, our solution is found to coincide with that of Horne and Horowitz [8] for a particular radius of the compact sigma model manifold.

A previously constructed [5] coupling of a Kahler sigma model to  $N = (2, 0)$ ,  $AdS_3$  supergravity differs from our model significantly in that the scalar fields of that model are neutral under the  $R$ -symmetry group  $U(1)$ . One consequence of having assigned a  $U(1)$  charge to the scalar fields is the emergence of a potential, which plays a significant role in the determination of our black string solutions.

As for the classical solutions of our model, it is natural to seek supersymmetric black holes. Indeed, we have searched for solutions of the form  $ds^2 = -e^{2A}dt^2 + e^{2B}dr^2 + r^2d\varphi^2$ . Setting the scalar field equal to zero reduces the equations of motion to those of pure anti de Sitter supergravity which is known to have the BTZ black hole solution [5]. However, if we insist on non-vanishing scalar fields, then the supersymmetry condition, together with the field equations, leads to a solution which upon coordinate transformations can be brought to the string solution of the form (4.24).

It would be interesting to find a solution in which the Maxwell field plays a role. In this context, we note that the Einstein's equation rule out nonvanishing gauge fields if we take all the fields to be only  $r$ -dependent. Presumably, therefore, one should allow  $x$  dependence as well.

A natural extension of our model would be the introduction of higher than  $N = (2, 0)$  supersymmetry. An interesting case to consider is the matter coupled  $N = (4, 4)$ ,  $AdS_3$  supergravity model which arises from the compactification of the  $D = 6$ ,  $N = (2, 0)$  supergravity coupled to  $n$  tensor multiplets, which has its origin in the  $K_3$  compactification of Type IIB string.

The question of how the models we have presented in this paper can be obtained from any compactification of M-theory or, for that matter, any higher dimensional supergravity theory remains open. The exact form of the CFT dual of our model formulated on the boundary of  $AdS_3$  also remains to be found.

Finally, we note that the model constructed in this paper does not involve any two-form potentials. These potentials would not describe propagating degrees of freedom but they might be useful in producing the low energy supergravity theory in the bulk in a form which is more appropriate in the string theory or the boundary CFT context. In the models we have constructed here, it is natural to introduce  $(n + 2)$  two-form potentials which form an  $SO(n, 2)$  vector. The form of the scalar potential in the action is then expected to change, but elimination of the two-form potential from the action through its equations of motion should yield the scalar potential of the model presented here.

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## References

- [1] J.M. Maldacena, *The large  $N$  limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2** (1998) 231, hep-th/9711200.
- [2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, Phys. Lett. **428B** (1998) 105.
- [3] E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. **2** (1998) 253.
- [4] A. Achucarro and P.K. Townsend, *A Chern-Simons action for three-dimensional AdS supergravity theories*, Phys. Lett **180B** (1986) 89.
- [5] J.M. Izquierdo and P.K. Townsend, *Supersymmetric spacetimes in (2+1) AdS-supergravity models*, Class. Quant. Grav. **12** (1995) 895, gr-qc/9501018.
- [6] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69** (1992) 1849, hep-th/9204099.
- [7] E. Witten and J. Bagger, *Quantization of Newton's constant in certain supergravity theories*, Phys. Lett. **115B** (1982) 202.
- [8] J.H. Horne, G.T. Horowitz, *Exact black string solutions in three dimensions*, Nucl. Phys. **B368** (1992) 444, hep-th/9108001.
- [9] J. Maldacena and A. Strominger, *AdS<sub>3</sub> black holes and a stringy exclusion principle*, **JHEP** 9812 (1998) 005, hep-th/9804085
- [10] S. Deger, A. Kaya, E. Sezgin and P. Sundell, *Spectrum of  $D = 6$ ,  $N = 4b$  supergravity on  $AdS_3 \times S^3$* , Nucl.Phys. B536 (1998) 110, hep-th/9804166.
- [11] J. de Boer, *Six-dimensional supergravity on  $S^3 \times AdS_3$  and 2d conformal field theory*, hep-th/9806104.
- [12] N. Marcus and J.H. Schwarz, *Three-dimensional supergravity theories*, Nucl. Phys. **B228** (1983) 145.
- [13] B. de Wit, A.K. Tollsten and H. Nicolai, *Locally supersymmetric  $D = 3$  nonlinear sigma models*, Nucl. Phys. **B392** (1993) 3, hep-th/9208074.
- [14] P.S. Howe, J.M. Izquierdo, G. Papadopoulos and P.K. Townsend, *New supergravities with central charges and killing spinors in 2+1 dimensions*, Nucl. Phys.**B467** (1996) 183, hep-th/9505032.
- [15] L. Mezincescu, P.K. Townsend and P. van Nieuwenhuizen, *Stability of gauged  $D = 7$  supergravity and the definition of masslessness in AdS in seven dimensions*, Phys. Lett. **143B** (1984) 384.
- [16] S.W. Hawking and G.T. Horowitz, *The gravitational Hamiltonian, action, entropy and surface terms*, gr-qc/9501014, Class. and Quantum Grav. **13** (1996) 1487.

- [17] G.W. Gibbons, C.M. Hull and N.P. Warner, *The stability of gauged supergravity*, Nucl. Phys. **B218** (1983) 173.
- [18] M.J. Duff, H. Lu, C.N. Pope and E. Sezgin, *Supermembranes with fewer supersymmetries*, hep-th/9511162, Phys. Lett. **B371** (1996) 206.
- [19] J.H. Horne, G.T. Horowitz and A.R. Steif, *An equivalence between momentum and charge in string theory*, Phys. Rev. Lett. **68** (1992) 568, hep-th/9110065.